Consider the linear:

Max Z = 5 X_1 + 12 X_2 + 4 X_3

Subject to:

X_1 + 2 X_2 + X_3 ≤ 10

2 X_1 - X_2 + 3 X_3 = 8

X_1 , X_2 , X_3 ≥ 0

1- **Find its Dual Problem.**
2- **Solve The Two Problems with Primary simplex.**

Solution

Standard Form:

Max Z = 5 X_1 + 12 X_2 + 4 X_3 + 0 S_1

X_1 + 2 X_2 + X_3 + S_1 = 10

2 X_1 – X_2 + 3 X_3 = 8

X_1 , X_2 , X_3 , S_1 ≥ 0

Dual:

Min Z = 10 Y_1 + 8 Y_2

Y_1 + 2 Y_2 ≥ 5	X_1
2 Y_1 – Y_2 ≥ 12	X_2
Y_1 +3 Y_2 ≥ 4	X_3
Y_1 ≥ 0	S_1

Y_2 is unrestricted,

2-Solve the Two Problems with Primary simplex.

Standard Form for Primal:

Max Z = 5 X_1 + 12 X_2 + 4 X_3 + 0 S_1

X_1 + 2 X_2 + X_3 + S_1 = 10

2 X_1 – X_2 + 3 X_3 = 8

X_1 , X_2 , X_3 , S_1 ≥ 0

Basic	X_1	X_2	X_3	S_1	R	Sol.	Ratio
Z	-5	-12	-4	0	M	0	
S_1	1	2	1	1	0	10	
R	2	-1	3	0	1	8	
Z	- 5 – 2 M	- 12 + M	- 4 -3 M	0	0	- 8 M	
S_1	1	2	1	1	0	10	10
R	2	-1	3	0	1	8	4
Z	0	-29/5	7/2	0	M + 5/2	20	
S_1	0	5/2	-1/2	1	-1/2	6	12/5
X_1	1	-1/2	3/2	0	1/2	4	----
Z	0	0	3/5	29/5	-2/5+M	54 4/5	
X_2	0	1	-1/5	2/5	-1/5	5/12	
X_1	1	0	7/5	1/5	2/5	26/5	

Optimal Solution:

$$Z^* = 54\ \frac{4}{5}$$

$$X_2^* = 5/12$$

$$X_1^* = 26/5$$

Standard Form For Dual:

Min W = 10 Y_1 + 8 Y_2^+ - 8 Y_2^- – S_1 – S_2 – S_3 – S_4 + MR_1 + MR_2 + MR_3

Y_1 + 2 Y_2^+ - 2 Y_2^- – S_1 + R_1 = 5

2 Y_1 – Y_2^+ + Y_2^- – S_2 + R_2 = 12

Y_1 +3 Y_2^+ - 3 Y_2^- – S_3 + R_3 = 4

$Y_1 \geq 0$

$Y_2 \geq -M$

$Y_1 , Y_2^+ , Y_2^- , S_1 , S_2 , S_3 , S_4 , R_1 , R_2 , R_3 \geq 0$

Basic	Y_1	Y_2^+	Y_2^-	S_1	S_2	S_3	R_1	R_2	R_3	Sol.
W	-10	-8	8	0	0	0	-M	-M	-M	0
R_1	1	2	-2	-1	0	0	1	0	0	5
R_2	2	-1	1	0	-1	0	0	1	0	12
R_3	1	3	-3	0	0	-1	0	0	1	4
W	-10+4M	-8+4M	8-4M	-M	-M	-M	0	0	0	21M
R_1	1	2	-2	-1	0	0	1	0	0	5
R_2	2	-1	1	0	-1	0	0	1	0	12
R_3	1	3	-3	0	0	-1	0	0	1	4
W	-22/3 +8/3M	0	0	-M	-M	-8/3 +1/3M	0	0	8/3 -4/3M	32/3 +47/3M
R_1	1/3	0	0	-1	0	3/2	1	0	-2/3	7/3
R_2	7/3	0	0	0	-1	-1/3	0	1	1/3	40/3
Y_2^+	1/3	1	-1	0	0	-1/3	0	0	1/3	4/3
W	0	22-8M	-22+8M	-M	-M	-10+3M	0	0	10-4M	40+5M
R_1	0	-1	1	-1	0	1	1	0	-1	1
R_2	0	-7	7	0	-1	2	0	1	-2	4
Y_1	1	3	3	0	0	-1	0	0	1	4
W	0	0	0	-M	-22/7 +1/7M	-26/7 +5/7M	0	22/7 -8/7M	26/7 -12/7M	368/7 +3/7M
R_1	0	0	0	-1	1/7	5/7	1	-1/7	-5/7	3/7
Y_2^-	0	-1	1	0	-1/7	2/7	0	1/7	-2/7	4/7
Y_1	1	0	0	0	-3/7	-1/7	0	3/7	1/7	40/7
W	0	0	0	-26/5	-12/5	0	26/5 - M	12/5 - M	-M	54 4/5
S_3	0	0	0	-7/5	1/5	1	7/5	-1/5	-1	3/5
Y_2^-	0	-1	1	2/5	-1/5	0	-2/5	1/5	0	2/5
Y_1	1	0	0	-1/5	-2/5	0	1/5	2/5	0	29/5

$W^* = 54\ \frac{4}{5}$ $S_3^* = 3/5$ $Y_2^{-*} = 2/5$ $Y_1^* = 29/5$

Best Wishes